Chaotic behaviour of the Caputo derivative

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We analyse the dynamical behaviour of the Caputo fractional derivative. Based on the Mittag-Leffler reproducing kernel space [6], we present the Mittag-Leffler-Caputo space, and prove that the complex Caputo derivative is well-defined and chaotic in such a space. This result seems to be consistent with the fact that the derivative is chaotic when considered as an operator in the space of entire functions [2].

In [1] the authors proved that different iterates of the derivative operator are disjoint hypercyclic. In this work, we show that different iterates of the complex Caputo derivative are d-hypercyclic.

Finally, continuing with the work initiated in [4] we provide sufficient conditions that ensure chaos for the most common numerical approximation of the Caputo derivative, that is, its L1 discretization.

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