Simultaneous bifurcation of limit cycles for Piecewise Holomorphic systems

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1 Abstract

Let $\dot{z} = f(z)$ be a holomorphic differential equation with center at p. In this work we are concerned about studying the piecewise perturbation systems $\dot{z} = f(z)+\epsilon R^{\pm}(z,\overline{z})$, where $R^{\pm}(z,\overline{z})$ are complex polynomials defined for $\pm \operatorname{Im}(z) > 0$. We provide an integral expression, similar to an Abelian integral, for the period annulus of p. The zeros of this integral control the bifurcating limit cycles from the periodic orbits of this annular region. This expression is given in terms of the conformal conjugation between $\dot{z} = f(z)$ and its linearization $\dot{z} = f'(p)z$ at p. We use this result to control the simultaneous bifurcation of limit cycles of the two annular periods of $\dot{z} = \mathrm{i}(z^2-1)/2$, after both complex and holomorphic piecewise polynomial perturbations. In particular, as far as we know, we present the first example of non nested limit cycles for piecewise holomorphic systems.

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