

# On Lagrangian foliations invariant by the geodesic flow of manifolds without conjugate points

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We study geodesic flows on a compact manifold without conjugate points  $M$  (a generalization of manifolds of negative curvature that allows regions of positive curvature). The uniqueness of Lagrangian continuous foliations invariant by the geodesic flow is a well-known property in the hyperbolic case (negative curvature). The generalization to weaker metrics is known in some cases despite its importance and usefulness. If  $M$  satisfies the Visibility condition, its Green bundles are continuous and has a hyperbolic closed geodesic then we show Green bundles are tangent to the horospherical foliations. Moreover the horospherical foliations are the unique Lagrangian invariant continuous foliations of the unit tangent bundle. In our setting, there may exist strips of bi-asymptotic geodesics that breaks the uniform hyperbolicity hence the proof method. In our context, we shall use the minimality of the horospherical foliations, the hyperbolic structure of the hyperbolic closed geodesic, and the fact that Green bundles can be integrated to Lagrangian invariant continuous foliations. This work generalizes to higher dimension some results of Gelfert-Ruggiero and Barbosa-Ruggiero [1, 2].

[1] J. Barbosa, R. Ruggiero, *Nonlinearity* **20**, 497 (2007).

[2] K. Gelfert, R. Ruggiero, *Proceedings of the Edinburgh Mathematical Society* **62**, 61–95 (2019).