Using Physics Informed Neural Networks to Find Singularities in Fluid Dynamics

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The emergence of singularities in solutions to fluid dynamics equations represents one of the most profound open challenges in mathematical physics, prominently exemplified by the Millennium Prize problem concerning the 3D Navier-Stokes equations. Singularities mark the breakdown of classical solutions, often signifying physical phenomena such as shock formation, cusps, or topological transitions. While this problem remains unsolved for high-dimensional systems, even simpler one-dimensional models such as the inviscid Burgers equation continue to offer valuable insight into the mechanisms underlying singular behavior. The Burgers equation, despite its mathematical simplicity, models key physical processes including shock waves and turbulence, and serves as a canonical testbed for exploring singularities in nonlinear partial differential equations (PDEs).

Recent advances in machine learning have introduced Physics-Informed Neural Networks (PINNs) as a compelling framework for solving PDEs by embedding physical laws directly into the structure of neural network training. Unlike traditional numerical solvers, which may struggle with resolving fine-scale features near singularities due to discretization limitations, PINNs can approximate solutions over continuous domains and leverage self-supervised learning from physical constraints rather than large datasets. In this work, we apply the PINN framework to the inviscid 1-D Burgers equation to investigate its capacity to capture singular solutions and self-similar structures near blow-up points.

A key component of our approach involves transforming the Burgers equation into self-similar coordinates, which reformulate the PDE into a time-independent form. This significantly reduces the computational complexity and emphasizes universal features of singularity formation, allowing the solution to be described by a single variable "profile" function. The resulting self-similar equation is then approximated using a PINN, with careful attention paid to loss function design, optimizer strategies (combining Adam and LBFGS), and sampling techniques. Notably, we explore the dual role of the self-similar exponent λ both as a fixed hyperparameter and as a trainable variable, investigating its impact on solution behavior.

Our results demonstrate that PINNs are capable of recovering both smooth and non-smooth self-similar profiles of the Burgers equation with high accuracy. The network successfully distinguishes between stable and unstable solutions, offering a clear depiction of how singularities form and evolve in this context. These findings confirm the viability of PINNs as a flexible and physically consistent tool for analyzing singularities in nonlinear PDEs. Moreover, the methodology developed here paves the way for tackling more complex systems such as the Euler and Navier-Stokes equations, marking a promising step toward data-driven discovery in mathematical fluid dynamics.