Title: Dissipative Euler flows originating from circular vortex filaments

Abstract:

Vortex filaments are three-dimensional structures in which the vorticity field is concentrated along a curve. The main challenge is to understand how such structures evolve over time. One of the difficulties in analyzing vortex filaments lies in the fact that the initial velocity field on the filament is singular—in fact, it becomes infinite—as does the kinetic energy. This singular behavior makes the mathematical treatment of vortex filaments nontrivial. Despite this, the problem is of great practical importance and has been studied extensively by physicists and engineers for several decades due to its relevance in fluid dynamics and turbulence.

To begin, we introduce the main heuristics and known results concerning the behavior of vortex filaments in the context of the three-dimensional Euler equations. In particular, we highlight the connection between vortex filaments and the binormal flow, discuss the vortex filament conjecture, and present a brief overview of the current state of the art.

Next, we state our main theorem, which establishes the existence of weak solutions to the three-dimensional Euler equations with initial vorticity concentrated on a circular filament. The kinetic energy of these solutions becomes instantly finite and decreases for positive times. Furthermore, the vorticity remains concentrated in toroidal structure that thickens while propagating along the axis of symmetry. We also compare our result with the behavior of vortex rings in the three-dimensional Navier–Stokes equations.

Finally, we provide some technical insights into the proof of the main theorem. To handle the singular nature of the initial data, we employ convex integration techniques within a suitable time-weighted functional space. We begin by outlining the general strategy of convex integration and then focus on constructing a suitable subsolution. Key aspects include the ansatz for the vorticity, the choice of pressure, the computation of the velocity field in axisymmetric coordinates, the handling of the Reynolds stress, and the energy estimates for the subsolution, particularly in relation to the energy of the final solution.

This is based on the joint work with F. Gancedo and F. Mengual
https://arxiv.org/abs/2404.04250