Poster presentation. Book of abstracts

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Complex Interpolation of weighted Bessel potential spaces and the Riesz Fractional Gradient.

Abstract: Based on the Fractional Laplacian and utilizing Riesz potentials, Shieh and Spector introduced the Riesz fractional gradient, denoted as D^s. Since its introduction, D^s has become a powerful tool in the study of variational problems and partial differential equations. Nonlocal (or fractional) models have garnered renewed interest following these developments and currently represent a very active area of research. However, there exists ambiguity in the definition of the fractional gradient and the associated function spaces in the works of Shieh and Spector. This issue has been carried over into early studies. Our aim is to shed light on this problem by identifying the inaccuracies and clarifying the notation and existing results.

We will also explore the main properties of the function spaces arising from the Riesz fractional gradient, which turns out to correspond to the classical Bessel potential spaces. These are also complex interpolation spaces between classical Sobolev spaces. Through interpolation theory, we will derive their principal properties, including embedding theorems and compactness results. We will connect the localization of D^s as $s \to 0^+$ and $s \to 1^-$ with Milman's results on the localization of norms for interpolation spaces. Additionally, we present a new proof of the Fractional Rellich-Kondrachov Theorem for Bessel potential spaces, based on an estimation involving translations within the framework provided by the Riesz fractional gradient, inspired by the results obtained by Del Teso, Gómez-Castro and Vázquez for the Sobolev-Slobodeckij spaces.

Once established the full picture for the "classical" case, we study the Riesz fractional gradient on other settings, such as obtaining the optimal Sobolev embeddings into the Lorentz scale by means of the weak estimates for the Riesz potential. We also introduce the weighted Fractional Sobolev spaces, which turns out to be also complex interpolation spaces. Those spaces are important for the study of the regularity of the fractional operators as well as we could also address another problem that is still open (even for the classical gradient) which is understanding the properties of generalized Orlicz spaces that are weighted.