Minimal bifurcation diagram of torus flows

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It is commonplace to study bifurcations for families of surface flows, where very complicated diagrams can emerge, even if no chaos is possible. The question arises whether there are topological criteria that force at least some amount of structure on the bifurcation diagram. And if so, what are the simplest diagrams that can occur.

To this end, we aim to understand the full bifurcation diagrams for families on a torus of the form

$$\dot{\mathbf{x}} = \boldsymbol{\Omega} + \mathbf{f}(\mathbf{x}), \tag{1}$$

where $\boldsymbol{\Omega} \in \mathbb{R}^2$ are the parameters and $\mathbf{x} \in \mathbb{T}^2$. In [1] it was shown that even the simplest possible bifurcation diagram of such family would contain a certain list of bifurcation points and curves. In particular, it contains an H point, from which infinitely many parameter curves of homoclinic connection emerge. It was then conjectured that a family existed containing precisely the bifurcations in the list, and no more, thus exhibiting the simplest bifurcation diagram in the class (1). We have shown [2] that the family

$$\begin{aligned} \dot{x} &= \Omega_x - \cos 2\pi(y - 3/24) - \varepsilon \cos 2\pi x \\ \dot{y} &= \Omega_y - \sin 2\pi y - \varepsilon \sin 2\pi x, \end{aligned} \tag{\#}$$

for $\varepsilon > 0$ small enough exhibits the simplest bifurcation diagram.

This is joint work with Prof R. S. MacKay and Dr C. Baesens (University of Warwick, UK).

References

[1] C. Baesens and R. S. MacKay. "Simplest bifurcation diagrams for monotone families of vector fields on a torus". In: *Nonlinearity* 31.6 (2018), pp. 2928–2981. DOI: 10.1088/1361-6544/aab6e2.

[2] Claude Baesens, Marc Homs-Dones, and Robert S MacKay. "Example of simplest bifurcation diagram for a monotone family of vector fields on a torus". In: *Nonlinearity* 37.11 (2024), p. 115001. DOI: 10.1088/1361-6544/ad6b70.