Mittag-Leffler fractional integrals in stochastic models for neuronal dynamics

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Abstract

Motivated by the need to model some neurophysiological evidences not included in classical neuronal models, we construct stochastic models based on coupled fractional stochastic differential equations, with different fractional orders. Indeed, one of the main motivations is to model neuronal dynamics on different time-scales. We give explicit expressions of the process representing the voltage variation in the neuronal membrane. Numerical evaluations of the average behaviors of involved processes are presented to put in evidence the features of the proposed models.

In order to refine the theoretical setting, we focus on Mittag–Leffler (ML) fractional integrals involved in the solution processes of a more general system of coupled fractional stochastic differential equations. We introduce the ML fractional stochastic process as a ML fractional stochastic integral with respect toa standard Brownian motion. We provide some representation formulas of solution processes in terms ofMittag–Leffler fractional integrals and processes. Computable expressions of the mean functions and of the covariances of such processes are specifically given. The application in neuronal modeling is provided, and all involved functions and processes are specifically determined. Numerical evaluations are carried out and some results are shown and discussed.

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