# Collective Behaviour of Chaotic Hénon Particles with Short-range Spatial Interaction

Congcong Du | Beijing Normal University

The interactions between chaotic systems have long been a subject of significant interest in studying emergent collective behaviors. Previous studies have explored these interactions through partial-dimensional coupling, full-space coupling, or mediation by environmental variables[1, 2]. However, the dynamics arising from local interactions within chaotic particles remain largely unexplored. In this work, we focus on the H´enon Map with short-range spatial interactions, uncovering novel dynamic behaviors that differ significantly from those observed under purely homogeneous global coupling conditions.

To explore this system quantitatively, we utilized time-series analysis, phase plane exploration, bifurcation studies and the Largest Lyapunov Exponent (LLE) calculations. Both individual and collective behaviors were examined, leading to three key findings: (1) Phase Transitions in Dynamics. Increasing the interaction strength induces a sequence of state transitions, including chaos, period-2-oscillations, localized-chaos and synchronization. The localized-chaos state can be further classified into two distinct subtypes: irregular localized-chaos and regular localized-chaos. These states represent a quasi-stable balance between short-range spatial interactions and intrinsic chaotic dynamics, illustrating the coexistence of chaotic and synchronized regimes. (2) Permanent Group Formation. Starting from random initial conditions, particles spontaneously form small groups. While the time required for a particle to settle varies, once integrated into a group, particles remain in stable regions indefinitely, provided there are no external perturbations. (3) Group Mergers and Splits. When iteratively adjusting the interaction strength and using the final stable state of the preceding experiment as the initial condition for the next, groups exhibit merging and splitting behaviors over successive trials. The evolution of behaviors strongly depends on the direction of change in A, with decreasing A producing simpler dynamics, hinting at a hysteresis effect. Our results highlight the impact of local interactions on chaotic particles, with important implications for biology and neuroscience, providing fresh perspectives on swarm dynamics of complex systems, emergent spatio-temporal dynamic activities in large-scale brain networks, and the mechanisms underlying epilepsy and seizures.