Analysis of bursting in a next-generation neural mass model with spike-frequency adaptation using nonstandard geometrical singular perturbation theory

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Recently, Ferrara et al. [1] published a next generation neural mass model with spike frequency adaptation (SFA). The introduction of SFA in a single, fully connected quadratic integrate and fire (QIF) neural network, leads to the emergence of bursting in excitatory networks.

We aim to extend the slow-fast analysis that the authors performed on the single QIF neural mass model to understand the mechanisms underlying bursting. We show that standard slow-fast analysis does not suffice to explain the model dynamics, but that nonstandard geometrical singular perturbation theory (GSPT) [2] can be applied and provides insight. This approach is justified by the presence of a small additive parameter in the population firing frequency (r) equation, which represents the population's heterogeneity.

After applying a state-dependent time-scale transformation, we get a slow-fast system in nonstandard form. Taking the singular limit, we obtain the critical manifold $r = 0$. This manifold is attracting for negative mean voltage ($V < 0$) and repelling for $V > 0$. On $r = 0$, V increases and eventually becomes positive. The system then makes a rapid transition towards the subspace $V = 0$ and high r values.

For $V \approx 0$, we introduce the second order derivative of V , transforming the system into a perturbed Lienard system, which – for low values of the SFA variable (A) and neglecting the perturbation – has a center. When the system oscillates, the perturbation is constantly negative, which turns the center into a weakly attracting focus, leading to damped oscillations. This equilibrium moves down the r direction in the (V, r) phase plane for increasing A values. This is because the perturbation skews V towards negative values, so that r decreases as V makes its derivative negative. Eventually, the system enters a region at lower r values where the $V = 0$ subspace becomes repelling. The trajectory therefore leaves the vicinity of V = 0, and approaches r = 0 quickly. The above steps then repeat, closing the cycle and producing the burst pattern.

In summary, we analyzed a next generation neural mass model showing that standard slow-fast analysis cannot explain the full dynamics, but we successfully applied nonstandard GSPT and results on Lienard systems to gain insight into the burst mechanism.

References:

[1] Alberto Ferrara, David Angulo-Garcia, Alessandro Torcini, and Simona Olmi. Population spiking and bursting in next-generation neural masses with spikefrequency adaptation. Physical Review E, 107(2):024311, February 2023.

[2] Martin Wechselberger. Geometric Singular Perturbation Theory Beyond the Standard Form, volume 6 of Frontiers in Applied Dynamical Systems: Reviews and Tutorials. Springer International Publishing, Cham, 2020.

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