Optimal control over damped Oscillations via response curves | Kevin Martínez Anhom

Periodic sustained oscillations, such as neural spikes or cardiac rhythms, are represented in dynamical systems by limit cycles. Damped oscillations, on the other hand, find their dynamical counterparts in strong (hyperbolic) and weak (non-hyperbolic) stable foci. However, it is intuitive to think that, with the help of a carefully designed external input, one can force the spiraling trajectories around a stable focus to close onto a periodic orbit and thus, induce sustained oscillations in a damped oscillator.

In the work we present here, we introduce an Augmented Phase Reduction (APR) formalism around strong stable foci, analogous to the one defined for limit cycles (see [1] and [2]), that facilitates the study of the effects that external inputs may exert on the dynamics of this type of damped oscillations. Furthermore, based on this novel APR formalism, we successfully pose and solve an optimal control strategy (inspired by [3]) for the induction of a limit cycle around a strong stable focus using a minimum-energy external input.

We successfully applied this theory to the practical cases of a strong linear focus and the Fitzhugh-Nagumo model (see [4] and [5]). Positive results in the first case (see Figure 1A), regarded as an easy academical model, allowed an exhaustive analysis to find some interesting properties of the control algorithm. Positive results in the second case (see Figure 1B) proved our optimal control strategy to be effective in enhancing the oscillatory regime of the most realistic neuron models, making them excitable even for low-intensity external stimuli. This opens the door to potential practical and therapeutical applications of this technique.

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