Koopman analysis of stochastic oscillator networks | Pierre Houzelstein

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Abstract
Collective rhythms and node synchronicity are ubiquitous phenomena in neural circuits, and have been linked to important cognitive processes, such as speech and memory. As such, being able to determine whether a network of neurons is an a synchronous state from their activity is an important question.

The Kuramoto Order Parameter (KOP) has proven to be a fertile tool to study phase synchronization in networks. It maps the dynamics of the full network to the complex plane, allowing to detect transitions to and fro full phase synchrony and providing a mean collective phase value.

However, to compute the KOP, one must have a phase function for each node. Getting this phase function might be difficult when the nodes are stochastic, as real-world systems often are – even more so when the oscillations are noise-induced.

In this contribution, building upon the results in [1], we suggest that Koopman theory can be leveraged to provide an alternative path into the analysis of network synchrony.

Using a variant of Dynamic Mode Decomposition called ResDMD [2], we map the network dynamics onto the so-called Q-function, the complex eigenfunction of the Koopman operator K corresponding to the dominant metastable oscillatory mode of the dynamics. This projects the collective network activity onto a linear, complex oscillator whose dynamics is well understood.

Our approach leads to a description of the mean phase dynamics similar to that provided by the KOP, but without requiring the knowledge of the individual node phase functions. Additionally, we recover the Quality Factor (QF) which provides a quantitative measure of the robustness of the collective oscillation: a high QF indicates strong coherence. Finally, using a theoretical framework for stochastic phase reduction that we have recently developed, see [3], we can construct a one-dimensional stochastic differential equation (SDE) which approximates the evolution of the mean collective phase.

We will illustrate our approach with networks with individual nodes exhibiting the canonical Hopf and SNIC bifurcations, which are of particular relevance in neuroscience.

This approach shows promise as a new way to investigate oscillatory activity in neural data, especially given that Koopman/DMD research is a very active field attracting considerable interest.

References

[1] Pérez-Cervera et al., PNAS (2023).
[2] Colbrook et al., Nonlinear Dynamics (2023)
[3] Houzelstein et al., Physical Review Research (2025) – accepted for publication.