Classification of Neural Mass Models based on codimension-2 bifurcations | Gabriele Casagrande

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Neural mass models (NMMs) are mathematical frameworks widely used to describe the collective activity of neuronal populations, enabling the study of complex phenomena such as brain oscillations, pathological dynamics, and cognitive processes in a simplified yet biologically meaningful way.

Different qualitative regimes in the dynamics of these models are achieved by changing parameters value. Performing bifurcation analysis allows us to define different regions in the space of parameters characterized by distinct pattern of activity. This kind of study usually relies upon the identification of codimension-1 bifurcations, namely bifurcations that require tuning of one parameter. High-codimension bifurcations are less frequently discussed in the literature, both because they are more challenging to identify and because it is unlikely for a model to satisfy the conditions on the parameters required to encounter them. However, it has been highlighted that high-codimension bifurcations play the role of 'organizing centers' for the dynamics of a model, as the topology of the bifurcation diagram in their neighborhood is organized in a reliable way. The benefits of recognizing these bifurcations are manifold: it helps to more easily identify different dynamical regimes available in specific models, to identify characteristic structures common to distinct models, or to define a mapping from the normal form of a specific bifurcation, when available.

In this work we systematically review several NNMs, both phenomenological and biophysically inspired ones, to develop a classification

based on this correspondences between common behaviors and the presence of high-codimension bifurcations, thereby expanding on the approach proposed by Touboul and colleagues [1]. In particular, we start with planar codimension-2 bifurcations, for which previous studies have given interesting insights about the development of phenomenological models, accounting for reproducing behaviors observed in EEG recordings and different patterns of bursting activity [1, 2]. For instance, the co-existence of up states, corresponding to high firing rates, and down states, corresponding to low firing rates, in multiple neural mass models (e.g. Jansen-Rit, Montbrio [3]), can be related to the presence of a specific codimension-2 bifurcation, the cusp bifurcation, where two curves of saddle-node bifurcation intersect. Another dynamical feature common to various neural mass models is the co-existence of a silent and active spiking state. Models showing this characteristic, have been widely used in the literature to describe both healthy or pathological oscillatory activity in neural populations. Such behavior can be traced to the presence of specific codimension-2 bifurcations. In particular, the Bautin singularity and the saddle-node loop bifurcation, which organize several neural models and play
a key role those targeting epileptic seizures (e.g. Epileptor [4], Zeta model, Wendling-Chauvel).