On the nature of turbulence

Álamo Rodríguez, Luis[1]

luisalamo224@gmail.com, luisalamo@correo.ugr.es

[1]University of Granada
[2]University of Oviedo

Abstract

In contrast to the laminar regime in which the movement of a fluid takes place over independent parallel layers, the turbulent regime occurs when, due to sudden changes in their physical properties –velocity and pression, mainly–, these layers intermingle. Turbulent fluids are subject to highly irregular and irreversible conditions, so that small perturbations of these flows lead to completely different looking behaviours. Therefore, we are facing a manifestation of chaos.

The particular problem we address in this paper concerns the genesis of turbulence, we ask ourselves about the nature and evolution of this concept. To this end, the main objective is to make a historical review of the different proposals that have emerged over the last century, assessing its scope from the perspective of dynamical systems theory.

We start from Landau's pioneering proposal in 1944 [1]. Briefly, Landau's proposal is based on the assumption that the physical parameters $\mathbf{x}(t)$ describing the motion of a turbulent flow are quasi-periodic functions of time. It turns out that a quasi-periodic flow of Landau's form, whose image is a k-dimensional torus $\mathbb{T}^k$, is not structurally stable. In other words, quasi-periodic flows does not last over time, so it would be impossible to observe turbulence in nature. Consequently, when $t \to \infty$, an integral curve $\mathbf{x}(t)$ could be attracted by an equilibrium point, by a periodic orbit, or by some more general attractor type (a strange attractor).

On the other hand, Ruelle and Takens' proposal (1971) [3] for the nature of turbulence is aimed at finding strange attractors in a dissipative system such as a viscous fluid. In this sense, a path towards the genesis of invariant tori for a uniparametric family of fields is given. We then see that a small perturbation of a quasi-periodic flow in a torus gives flows with strange attractors in dimension $k > 2$. Although the approach of Ruelle and Takens has had numerous successes in low dimension even at the experimental level, we will justify that it has limited impact on the study of turbulence.

We end this paper with one good way to illustrate qualitatively the transition to turbulence of a viscous flow: the Taylor-Couette flow.

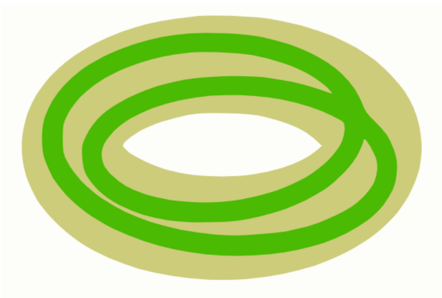

Figure 1: Smale's solenoid defined in the solid torus $\Pi = \mathbb{S}^1 \times \overline{D}$, with $\overline{D} = \{z \in \mathbb{C} : |z| \leq 1\}$.

Acknowledgement

The author would like to acknowledge J.A. Rodríguez Méndez for supervising and contributing to this work.

References

[1] L. Landau *On the problem of turbulence*, Collected Papers of L.D. Landau, pp. 387-391, 1965.

[2] L. Mora, M. Viana *Abundance of strange attractors*, Acta Mathematica 171.1, 1963.

[3] D. Ruelle, F. Takens *On the nature of turbulence*, Communications in Mathematical Physics 20.3, pp. 167-192, 1971.

[4] S. Smale *Differentiable dynamical systems*, Bulletin of the American Mathematical Society 73.6, pp. 747-817, 1967.