Incompleteness of recurrent network dynamics:
theoretical limitations to computation on neuronal networks

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A network of neurons is expected to be able to carry out a variety of complex computations, and indeed recurrent neuronal networks can be shown to be universal function approximators and to be Turing-complete. However, these results require that the neurons can be connected in precise ways, specific to each application.

This study shows that, starting from an arbitrary network, there are limitations on the possible network dynamics, as well as constraints on the possible network topologies that can be obtained through activity-dependent alterations of the network connectivity.

By studying the dynamics close to a stable state, it is shown that computations on a fixed network are limited by the internal dynamics of the constituent elements: perturbations along some eigenvectors are exponentially damped, thus small inputs differing along these directions cannot be distinguished after a short transient. This shows that the representational capacity of the dynamics on networks is incomplete: there are certain perturbations composed of a finite combination of the eigenvectors of the system for which it cannot be determined whether they have occurred.

Moreover, it can be shown that for common correlation-based synaptic plasticity mechanisms the sign of the growth-rate of the eigenvalues depends on the growth-rate of the perturbations about the stable state. Thus this type of plasticity alone cannot salvage the situation by strengthening the damped eigenvectors in order for them to become unstable.

Finally, some implications of these results, in both negative and positive sense, for computation and functioning of neuronal systems will be discussed.

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