Microscopic organization of higher-order networks drives explosive collective behaviors [1]

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In the recent years, network science has put the focus on the development of a general framework comprising group interactions for the study of collective behaviors well beyond the limitations of pairwise interactions. In particular, it has provided a natural pathway towards explosive transitions [2], with examples spanning social contagion, synchronization or game theory. In this work [1] we show that the sole presence of higher-order interactions could not be sufficient to lead to explosive transitions. What is also crucial is the way in which the nodes interact in groups, and how many nodes of a group are also present in other groups of the same size. To quantify this, we introduce the intra-order hyperedge overlap $\mathcal{T}^{(m)}$, a metric which quantifies the overlap among the hyperedges of a higher-order network by evaluating the number of nodes shared among the different hyperedges of the same order m. The metric is well bounded between $\mathcal{T}^{(m)} = 0$ when there is no overlap, and $\mathcal{T}^{(m)} = 1$ when the overlap is maximum. Equipped with this metric, we first study hypergraphs describing higher-order interactions in various real complex systems, showing that they exhibit a large variety of values of intra-order hyperedge overlap. We then investigate if and how the different level of intra-order hyperedge overlap of a system affects the emergence and properties of its collective behavior. In order to do so, we generate a set of synthetic regular structures for all the set of intra-order hyperedge overlap values $\{\mathcal{T}^{(2)}\}$. We focus on two radically different dynamical processes, namely social contagion and synchronization of coupled dynamical systems. The social contagion dynamic is modelled as a higher-order SIS model where the transition from susceptible S to infectious I can happen via $1-$hyperedges with a probability $\beta^{(1)}$, or via $2-$hyperedges with a probability $\beta^{(2)}$. As in the standard SIS model, an infected individual can recover with probability μ, and we define the rescaled infectivities as $\lambda^{(m)} = \beta^{(m)}/\mu$. The order parameter is the stationary fraction of infectious population $\rho^\star$. The second dynamic is a higher-order Kuramoto model with a different coupling term for each interaction order. In our case, we have $1-$hyperedges coupling with strength $\sigma^{(1)}$ and $1-$hyperedges coupling with strength $\sigma^{(2)}$, and the order parameter $\langle r \rangle$ measures the degree of synchronization. In Fig. 1, we show that hypergraphs with low intra-order hyperedge overlap undergo explosive transitions, characterized by a bistable region where both an active/synchronized state and an absorbent/incoherent state coexist. Conversely, hypergraphs with a intra-order hyperedge overlap larger than a critical value can only exhibit continuous transitions. The similitude between both phase diagrams in Fig. 1.(a) and Fig. 1.(b) highlight the universal effect generated by intra-order hyperedge overlap in higher-order structures. In particular, these results reveal that the structural organization of hyperedges shapes the way collective behaviors emergence in systems with higher-order interactions.

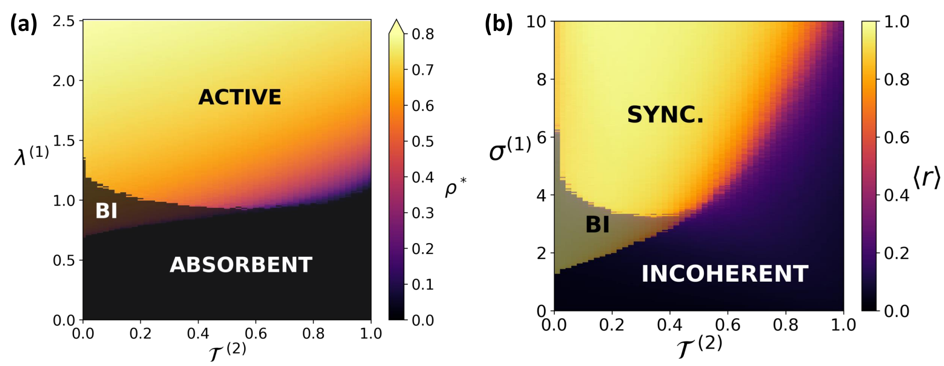

FIG. 1. **Universal effect of intra-order hyperedge overlap on emergent phenomena.** (a) Phase diagram for the SIS model. Three phases emerge as a function of $\lambda^{(1)}$ and of the hyperedge overlap $\mathcal{T}^{(2)}$: an absorbent phase with $\rho^\star = 0$, an active phase with an endemic stationary state $\rho^\star \neq 0$, and a bistability phase, where the stationary state depends on the initial conditions. (b) Phase diagram for the Kuramoto model. Three phases emerge as a function of $\sigma^{(1)}$ and $\mathcal{T}^{(2)}$: an incoherent phase with low values of $\langle r \rangle$, a synchronized phase with large $\langle r \rangle$, and a bistability phase where the system can be synchronized or not depending on the initial conditions.

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[1] F. Malizia, S. Lamata-Otín, M. Frasca, V. Latora, and J. Gómez-Gardeñes, arXiv preprint arXiv:2307.03519 (2023).
[2] F. Battiston, E. Amico, A. Barrat, G. Bianconi, G. Ferraz de Arruda, B. Franceschiello, I. Iacopini, S. Kéfi, V. Latora, Y. Moreno, et al., Nature Physics **17**, 1093 (2021).