Title:

Infinitesimal and tangential 16-th Hilbert problem on zero-cycles

Abstract:

In this paper, given two polynomials f and g of one variable and a **0**-cycle C of f, we consider the deformation $f+\epsilon g$. We define two functions: the displacement function $\Delta(t,\epsilon)$ and its first order approximation: the abelian integral $M1(t)$. The infinitesimal and tangential 16-th Hilbert problem for zero-cycles are problems of counting isolated regular zeros of $\Delta(t,\epsilon)$, for ϵ small, or of $M1(t)$, respectively. We show that the two problems are not equivalent and find optimal bounds, in function of the degrees of f and g, for the infinitesimal and tangential 16-th Hilbert problem on zero-cycles. These two problems are the zero-dimensional analogue of the classical infinitesimal and tangential 16-th Hilbert problems for vector fields in the plane.