A pragmatic approach to predict tipping points in dynamical systems

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All dynamical systems in nature display transient dynamics, where a system experiences a regime shift after crossing a tipping point or critical threshold (Scheffer 2009; Strogatz 2015). The emergence of marine photosynthetic bacteria 2.5 billion years ago triggered a transition from low to high oxygen in the atmosphere (Margulis and Sagan 1997; Lyons et al. 2014), and the disruption of the thermal-haline marine current in the Atlantic 12000 years ago (the Younger-Dryas period) reversed the recovery from the last Glaciation period (Scheffer 2009; Cheng et al. 2020). Even the periodic dance of planets around the Sun is predicted to derive into a chaotic dynamic in two billion years, likely leading to a collision between Earth, Mars, Venus, or Mercury (Laskar and Gastineau 2009; Hayes 2007), although by then the Sun will have expanded as a red star and will likely engulf the rocky planets (Schroder and Smith 2008; De et al. 2023). Transient dynamics are, thus, unavoidable in the study of natural systems. However, it is very difficult to predict the tipping points causing a regime shift(Seekell 2016; Scheffer et al. 2009).

Here, we introduce a new theoretical framework to predict tipping points. The approach builds on two assumptions. First, a process in a dynamical system is considered active if it changes substantially within a finite observational timescale that depends on the specific study of interest. Second, the substantial change is normalized to a reference value of the variable impacted by the process. As a consequence, processes governing the dynamics of a system can become inactive over an observation time, and tipping points are reached when the processes switch off or on. These two principles were encoded mathematically, defining dynamic weights: contributions relative to reference values of the dynamic processes to the rate-of-change of agents in the system for a finite observational time. Operationally, processes were considered active if their weights were above a critical threshold. Tipping points were defined as reaching critical thresholds that activated or inactivated processes. This Finite Observational Dynamics Analysis Method (FODAM) method predicted that a system with n underlying dynamic processes could display 2^n different dynamic regimes.

We illustrate the application of the general method using a classic predator-prey system modeled by a Lotka-Volterra dynamical system. To make the choice of conditions more specific, we focused on the interaction of bacteria and bacteriophages. The approach identified 16 (2^4) dynamic regimes depending on which processes were active within the observational timescale of the study. The critical thresholds obtained were also applied to investigate the resilience of the system to perturbations, showing a dramatically different response when perturbations crossed these values. The discussion section further elaborates on how to use this approach to investigate real systems with many variables. We also elaborated on how the assumptions lead to bounded theoretical errors that can be reduced based on the needed experimental resolution.

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