

The D -Mercator method for the multidimensional hyperbolic embedding of real networks

Robert Jankowski^{1,2}, Antoine Allard^{3,4}, Marián Boguñá^{1,2} & M. Ángeles Serrano^{1,2,5}

¹Departament de Física de la Matèria Condensada,

Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain

²Universitat de Barcelona Institute of Complex Systems (UBICS), Universitat de Barcelona, Barcelona, Spain

³Département de physique, de génie physique et d'optique, Université Laval, Québec (Québec), Canada G1V 0A6

⁴ Centre interdisciplinaire en modélisation mathématique, Université Laval, Québec (Québec), Canada G1V 0A6

⁵ ICREA, Passeig Lluís Companys 23, E-08010 Barcelona, Spain

One of the pillars of the geometric approach to networks has been the development of model-based mapping tools that embed network topologies in their latent geometry. In particular, Mercator [1] embeds real networks into the hyperbolic plane. However, some real networks are better described by the multidimensional formulation of the underlying geometric model [2]. Here, we introduce D -Mercator [3], an embedding method that goes beyond Mercator to produce multidimensional maps of real networks into the $D + 1$ hyperbolic space where the similarity dimension is represented in a D -sphere. We evaluated the quality of the embeddings using synthetic $\mathbb{S}^D$ networks. We also produced multidimensional hyperbolic maps of real networks that provide more informative descriptions than their two-dimensional counterparts and reproduce their structure more faithfully. Having multidimensional representations will help to reveal the correlation of the dimensions identified with factors known to determine connectivity in real systems and to address fundamental issues that hinge on dimensionality, such as universality in critical behavior. D -Mercator also allows us to estimate the intrinsic dimensionality of real networks in terms of navigability and community structure, in good agreement with embedding-free estimations.

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