Quadratic tangencies in the RPC4BP

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January 28, 2024

Abstract

The restricted planar circular 4-body problem (RPC4BP) models the motion in the plane of a body of infinitesimal mass under the Newtonian gravitational field created by other three bodies called the primaries, which revolve around their common center of mass in circles. We assume that the masses of the three primaries are $\mu_1, \mu_2, 1-\mu_1-\mu_2$.

The dynamics of the system can be described by a time-dependent Hamiltonian of two degrees of freedom.

After introducing polar coordinates, performing a rescaling and restricting to a level of energy of an associated autonomous Hamiltonian, we consider a transversal section to the flow and the corresponding Poincaré map. This map has a parabolic fixed point which possesses stable and unstable one-dimensional invariant manifolds: γ^s and γ^u.

By analyzing the so-called Melnikov function, which gives an approximation of the difference between the invariant manifolds, we prove the existence of quadratic tangencies between γ^s and γ^u when $\mu_1 = \mu_2 = \mu \approx 1/3$.

In the case of a hyperbolic point, a quadratic tangencies which unfolds generically (which essentially means that the invariant curves have non-zero relative velocity when we change the parameter μ) gives rise to what is called Newhouse phenomena. Since our fixed point is not hyperbolic but parabolic, we have to overcome some difficulties, related to the lack of a λ-lemma and the problem in defining Shilnikov variables ir our setting, for which we resort to normal form methods. Also a renormalization argument used in the classical result needs to be adapted.