DYNAMICS OF QUANTUM PUBLIC GOODS GAMES

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ABSTRACT

The concept of overusing common goods was first discussed by W. Foster Lloyd in 1832 [1]. He highlighted the distinction between the saturation points of individual and communal resources, using the example of cattle and pasture. The problem of population growth, rather than mere resource allocation, was at the core of his argument. G. Hardin's influential 1968 article [2] coined the term 'tragedy of the commons,' metaphorically emphasizing how free access to finite resources leads to overexploitation.

Building on this foundation, extensive research has analyzed public goods management, often modeling it as a game (PGG). Some researchers use the term Common-Pool Resources (CPR) games when players extract or collect from a limited common resource. Many studies model the CPRG as a discrete game, typically a prisoner’s dilemma game , reflecting the choice between cooperation and defection.

In this work, we explore the dynamics of CPRG from a continuous game perspective, where two players consume a common resource. Using Cournot-type payoff functions, we limit the total resource intake to its maximum capacity. The unitary marginal benefit becomes zero if the common resource limit (Gmax) is exceeded (tragedy). We introduce a parameter *k* to control the model's elasticity and analyze the impact of entanglement between players (quantum game) [3,4].

The behaviour of the dynamics is studied considering how the fixed points (particularly the Nash equilibrium) and the stability of the system vary depending on the different values of the parameters involved in the model. In the analysis of this game, it is especially relevant to consider the extent to which the resource is exploited, since the output of the players is highly affected by this issue. It is studied in which cases the resource can be overexploited, adjusting the parameters of the model to avoid this scenario when it is possible. The results are obtained from an analytical point of view and also graphically using bifurcation diagrams to show the behaviour of the dynamics.

We have observed that entanglement decreases the stability of the system, i.e., the higher the entanglement is, the lower the stability in terms of the speed of adjustment is. Another important result is that the stability zone does not depend on the parameters k and Gmax.

We conclude that control over common resource saturation and prevention of overexploitation can be achieved through the modulation of entanglement and *k*. Notably, overexploitation may occur in the chaos zone, even if absent in the stable zone. Comparatively, the quantum game, when contrasted with the classic game, aids in controlling resource saturation and improving payoffs by fine-tuning entanglement and *k*. Achieving system optimization necessitates balancing the negative impact of entanglement on stability.

References

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