The dynamics of coupled logistic maps

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We consider the system of difference equations

$$\begin{cases} x_{n+1} = (1-\varepsilon) f_\mu(x_n) + \varepsilon f_\mu(y_n), \\ y_{n+1} = \varepsilon f_\mu(x_n) + (1-\varepsilon) f_\mu(y_n), \end{cases} \tag{1}$$

where $\varepsilon \in [0,1]$ is a coupling parameter and $f_\mu(x) = \mu x(1-x)$, $\mu \in (0,4]$, $x \in [0,1]$, is the classical logistic family widely studied [5]. This model was proposed and studied in [3, 4] and it is motivated by difference equations associated to the Belousov-Zhabotinsky chemical reaction [1]. This model is included in the family of the so-called dynamics of coupled map lattices (see e.g. [2]). We study the synchronization properties of this family of maps and solve an open question posed in [3] by proving that the dynamics of the model need not be lying on the diagonal set $\Delta = \{(x,x) : x \in \mathbb{R}\}$.

References

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