Title:

Principal Bautin ideal of monodromic singularities with inverse integrating factors

Abstract:

We analyze the structure of the Poincaré map π associated to a monodromic singularity of an analytic family of planar vector fields. We work under two assumptions. The first one is that the family possesses an inverse integrating factor that can be expanded in Laurent series centered at the singularity after a weighted polar blow-up fixed by the Newton diagram of the family. The second one is that we restrict our analysis to a subset of the monodromic parameter space that assures the non-existence of local curves with zero angular speed. The conclusions are that the asymptotic Dulac expansion of π does not contain logarithmic terms, indeed it admits a formal power series expansion with a unique independent generalized Poincaré-Lyapunov quantity, which can be computed under some explicit conditions. Moreover we also give conditions that guarantee the analyticity of π, in which case we show that the Bautin ideal is principal and therefore the cyclicity of the singularity with respect to perturbation within the family is zero.