Beliefs and Social Networks: Exploring Opinion Dynamics in Interconnected Complex Systems

Irene Ferri[1], Albert Díaz-Guilera[1], and Hiroki Sayama[23]
[1]Departament de Física de la Matèria Condensada and Institute of Complex Systems (UBICS)
Universitat de Barcelona, 08028 Barcelona, Spain
[2]2Center for Collective Dynamics of Complex Systems, Binghamton University, Binghamton, NY 13902-6000, USA
[3]Waseda Innovation Lab, Waseda University, Shinjuku, Tokyo 169-8050, Japan

We consider a system composed of $i = 1, N$ agents embedded in a fully connected graph, where links represent social ties. Each agent possesses an internal network consisting of n beliefs on different topics labeled as $\mu = A, B, C, \ldots$. Beliefs sharing the same label form a complete graph, but there are no connections between beliefs with different labels belonging to different agents (see Fig **??**(Top)). We assume that beliefs can internally influence each other, and that are connected forming one of the following three structures:

- Complete graph: All beliefs within an agent are interconnected.
- Ring: An example of a very sparse belief system.
- Star: One core belief, and all other beliefs are attached to it.

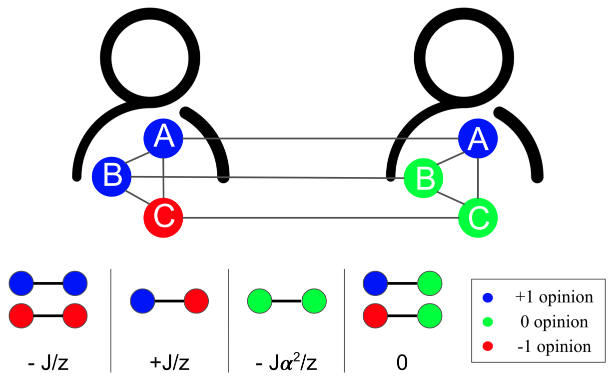

Fig. 1. (Top) Illustrative scheme of two connected agents with three internal beliefs A, B, and C forming a triangle. (Bottom) Contributions to the total energy for every possible pair of connected beliefs. Colors represent the belief states. J is the coupling factor, z is the average number of connections (different for the external and the internal terms), and α is the neutrality parameter.

The total belief adjacency matrix can be represented by the Cartesian product of the external and internal networks: $A_{\text{ext}} \square A_{\text{int}}$. Beliefs can exist in one of the following states, represented by two-dimensional vectors (introduced in [**?**]):

- $\mathbf{S}_\mu = (1, 0)$; positive
- $\mathbf{S}_\mu = (0, \alpha)$; neutral
- $\mathbf{S}_\mu = (-1, 0)$; negative

Here, α is a dimensionless parameter, and the index μ refers to any belief in the system into any agent of the system. Interactions between pairs of connected beliefs aim to minimize the following Ising-like Hamiltonian:

$$H = -\frac{J}{z_{\text{ext}}} \sum_{\langle \mu,\nu \rangle_{\text{ext}}} \mathbf{S}\mu \cdot \mathbf{S}\nu - \frac{J}{z_{\text{int}}} \sum_{i=1}^{N} \sum_{\langle \mu,\nu \rangle_{\text{int}}} \mathbf{S}\mu \cdot \mathbf{S}\nu, \quad (1)$$

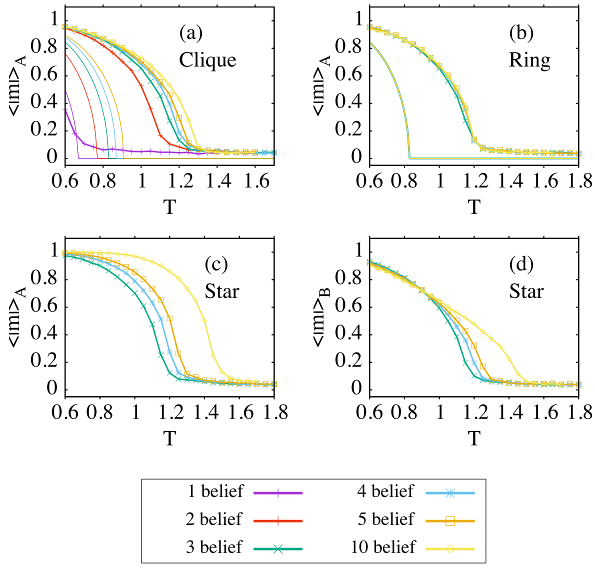

Fig. 2. Absolute average value for the magnetization of a particular belief in the stationary state as a function of the temperature T for a system of $N = 500$ agents with an internal belief network being (a) a clique, (b) a ring, (c) a star (core), and (d) a star (periphery). Line points correspond to results obtained by averaging 100 Metropolis Monte Carlo simulations, while continuous lines in (a) and (b) correspond to the annealed mean-field approximation.

Here, z_{ext} is the external number of connections $(N - 1)$ between agents, and z_{int} corresponds to the average internal number of connections, which depends on each internal topology. The first term of the Hamiltonian extends over connected beliefs in different agents, while the second term covers the internal connections of all agents.

We calculate the contribution to the energy of each pair of connected beliefs as the scalar product of the two opinion vectors, summing all contributions to compute the global internal energy (see Fig. 1(Bottom)). The positive coupling constant J is set to one for simplicity, but we normalize it by the average number of connections in each term to maintain comparability between internal and external contributions. As the Hamiltonian operates over nearest neighbors, its form reflects the idea that holding an opinion different from those of your connected peers has a cost, whereas agreement with neighbors decreases the system's energy asymmetrically for extremists and neutral agents. Having internally different connected belief states has also an energy cost, in this case, related to the phenomenon of cognitive dissonance. By regulating the parameter α, we investigate the effects of neutral beliefs on the state of the system.

We introduce temperature to the model to account for external social agitation and high inner cognitive dissonance. The stationary state and dynamics of the model are studied using analytical approaches and Monte Carlo simulations.

A significant increase in the critical point is observed by merely adding a second internal belief to the agents. Beyond this point, adding more beliefs changes the critical temperature, depending on the internal topology of the system (see Fig. 2).

Additionally, we perform Monte Carlo simulations near the critical temperature with a mix of 50 percent agents of two different internal topologies. We find that the same type of agents behaves differently depending on whether they are mixed with one or another type.

[1] A. Svekenson, and A. Swami, *Reaching consensus by allowing moments of indecision*, Sci. Rep. **5**, 14839 (2015).

[2] H.L-J. van der Maas, J. Dalege, and L. Waldorp *The polarization within and across individuals: the hierarchical Ising opinion model* J. Complex Netw. **8**, 2 (2020)

[3] I. Ferri, A. Díaz-Guilera, and M. Palassini, *Equilibrium and dynamics of a three-state opinion model*, (2022), arXiv:2210.03054 [cond-mat.stat- mech]