Restoring uniformity in hypergraph spectral centralities via the *uplift*

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Abstract

Over the past few years hypergraphs have been established as the successor of graph theory in the quest of modeling and analyzing systems consisting of units and interactions among them [1]. These mathematical structures, where the interactions are not limited to being pairwise between nodes, are as varied and feature-rich as challenging to analytically study, which is why many traditional network science concepts have yet to find a proper generalization to this new setting.

A flagship concept in network theory is that of centrality measures: determining the importance of nodes within a network [2]. A particularly important kind is that of spectral centralities, which make use of algebraic representations of the network (e.g. adjacency matrix), rendering them better suited for theoretical analysis as opposed to other brute-force measures (e.g. betweenness centrality). The simplest of them, the eigenvector centrality, is troublesome to define in the hypergraph setting, for one usually needs several adjacency tensors to describe the different orders of the hypergraph.

In 2019, Benson came up with three generalizations of the eigenvector centrality to uniform hypergraphs [3], i.e. those where all interactions are of the same order. This is a very restrictive condition, and in this talk I will explain how we can move away from it using a transformation we named the *uplift* [4], settling once and for all the definition of eigenvector centralities in general hypergraphs.

References

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