

“Impossible” pattern formation made possible by pulsed interactions

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Many physical, chemical or biological systems experience instabilities that create spatially periodic distributions of substances or energy out of a homogeneous initial state. Several mechanisms have been identified for such *pattern-forming* processes, being the most well-known the Turing mechanism, in which a fast-diffusing *inhibitor* substance controls the growth of a slow-diffusing *activator*, leading to modulated distributions. This process is usually modeled with reaction-diffusion equations, so that interactions between substances occur locally in space.

Another mechanism that has been identified in the context of vegetation patterns [1] and cluster crystals [2] is *competitive clustering*, in which biological competition or repulsive interactions lead to spatially periodic arrangements of substances or of clusters of particles. This mechanism is often modeled in terms of non-local equations, in which interactions occur at finite distances within an interaction range.

A timely question is to assess if such non-local interactions can be obtained from more fundamental models, in which the action-at-a-distance is just an approximate description of interactions of substances or signals that are released by the agents and act locally, but at time-scales that are very fast, justifying their elimination from the description.

Here [3, 4] we show that, under rather standard assumptions, such approximate description leads always to non-local effective descriptions for which pattern formation does not occur. However, when the release of substances or signals occurs in a pulsed, intermittent manner, pattern formation instabilities can occur. This pulsed or flashing signal dynamics creates a route to pattern formation alternative to the most studied ones arising from Turing-like mechanisms.

Our finding is obtained by studying a general activator-inhibitor (population-signal) model, where a biological population interacts through the release of harmful signals with a fine-scale dynamics which is explicitly modeled. For slow signal dynamics (with timescales similar to those of the population), we recover standard reaction-diffusion dynamics which, for a broad set of population and signal dynamics, does not exhibit Turing instability for any values of model parameters. For the same system dynamics, but with sufficiently fast flashing signals, the system can be described by a single integrodifferential equation, where the toxic effects are captured by a competitive nonlocal spatial interaction. In this limit, and for a suitable class of signal dynamics, spatial instability can occur leading to pattern formation. Thus, our approach identifies a novel mechanism of pattern formation, which relies on the fine-scale and fast dynamics of signals of pulsed character, and bridges the reaction-diffusion and the integrodifferential descriptions in adequate limit cases.

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- [1] R. Martínez-García, C. Cabal, J.M. Calabrese, E. Hernández-García, C.E. Tarnita, C. López, J.A. Bonachela, *Integrating theory and experiments to link local mechanisms and ecosystem-level consequences of vegetation patterns in drylands*, Chaos Solitons and Fractals **166**, 112881 (2023).
 - [2] J-B. Delfau, C. López, E. Hernández-García, *Active cluster crystals*, New Journal of Physics **19**, 095001 (2017).
 - [3] E.H. Colombo, C. López, E. Hernández-García, *Pulsed interaction signals as a route to biological pattern formation*, Physical Review Letters **130**, 058401 (2023).
 - [4] Eduardo H. Colombo, Ricardo Martínez-García, Justin M. Calabrese, Cristóbal López, Emilio Hernández-García *Pulsed interactions unify reaction-diffusion and spatial nonlocal models for biological pattern formation*, arXiv: <https://arxiv.org/abs/2311.07135>