THE (SECOND-)LARGEST COMPONENT IN SPATIAL INHOMOGENEOUS RANDOM GRAPHS

JOOST JORRITSMA

In a series of papers with Júlia Komjáthy and Dieter Mitsche, we uncover the deep relation between three types of connected components in a large class of supercritical spatial random graphs that includes long-range percolation, geometric inhomogeneous random graphs, and the age-dependent random connection model. Let $\mathscr{C}_n^{(1)}$ and $\mathscr{C}_n^{(2)}$ denote the largest and second-largest component in the graph restricted to a d-dimensional box/torus of volume n, and let $\mathscr{C}(0)$ be the component in the infinite graph that contains a vertex at the origin. We identify $\zeta \in (0,1)$ such that

$$\begin{aligned} \mathbb{P}\big(|\mathscr{C}_n^{(1)}| \le (1-\varepsilon)\mathbb{E}\big[|\mathscr{C}_n^{(1)}|\big]\big) &= \exp\big(-\Theta\big(n^{\zeta}\big)\big), \\ |\mathscr{C}_n^{(2)}| \big/ (\log n)^{1/\zeta} &= \Theta_{\mathbb{P}}(1), \\ \mathbb{P}\big(n \le |\mathscr{C}(0)| < \infty\big) &= \exp\big(-\Theta\big(n^{\zeta}\big)\big), \end{aligned} \tag{1}$$

as n tends to infinity. In words, there is a single exponent ζ (depending only on a few parameters of the model) that is guiding the speed of the lower tail of large deviations for the giant, the size of the second largest component, and the decay of the finite cluster size distribution of the origin. During the talk, I will explain intuition about the relation between the three quantities through the lens of an example. Time permitting, I discuss also the upper tail of large deviations of the giant component, i.e., $\mathbb{P}\big(|\mathscr{C}_n^{(1)}| \ge (1+\varepsilon)\mathbb{E}\big[|\mathscr{C}_n^{(1)}|\big]\big)$, and show that it behaves drastically different compared to the lower tail in (1).

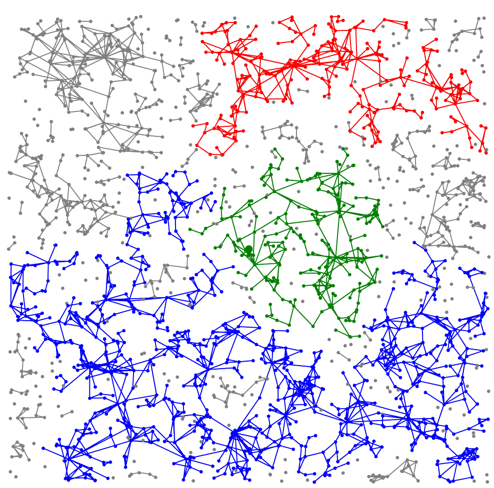

Figure 1. Simulation of a geometric inhomogeneous random graph (GIRG) restricted to a finite box. We unveil the relation between the size of the largest connected component (blue), the connected component containing the origin (green), and the size of the second-largest connected component (red) for a large class of graphs containing GIRGs.