Scale-invariant Harnack inequality for a class of nonlocal operators

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June 30, 2023

In this project, we consider a class of non-local operators on $\mathbb{R}^d$ having both diffusive and jumping parts of the following form

$$\begin{aligned}\mathcal{L}u(x) &= \frac{1}{2}\sum_{i,j=1}^{d} a_{i,j}(x)\frac{\partial^2}{\partial x_i \partial x_j}u(x) + \sum_{i=1}^{d} b_i(x)\frac{\partial}{\partial x_i}u(x) \\ &\quad + \int_{\mathbb{R}^d\setminus\{0\}} \Big(u(x+z) - u(x) - \nabla u(x)\cdot z\mathbb{1}_{\{|z|\le 1\}}\Big)\, n(x,dz). \qquad (0.1)\end{aligned}$$

Here the matrix $(a_{ij})(x)$ is uniformly elliptic, the vector $b(x)$ is bounded, and, for each fixed x, $n(x, dz)$ is a Lévy measure. Under some mild conditions, we establish the scale-invariant Harnack inequality for nonnegative $\mathcal{L}$-harmonic functions, and the Hölder regularity for $\mathcal{L}$-harmonic bounded functions. We also obtain analogous results for $\mathcal{L}$-caloric functions. Our approach is mainly probabilistic.This is the joint work with my advisor Zhenqing Chen.