Multi-frequency splitting of separatrices in Hamiltonian systems

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Abstract

We study the splitting of invariant manifolds of whiskered (hyperbolic) tori with two or three frequencies in nearly-integrable Hamiltonian systems whose hyperbolic parts are given by pendulums. We consider a 2-dimensional torus with quadratic frequencies or a 3-dimensional torus with a frequency vector given by a cubic irrational number whose two conjugates are complex (for instance, the real root of z^3+z-1). Applying the Poincaré-Melnikov method, we find exponentially small asymptotic estimates for the maximal splitting distance between the stable and unstable manifolds associated to the invariant torus, and we show that such estimates depend strongly on the arithmetic properties of the frequencies. The function in the exponent turns out to be periodic with respect to the perturbation parameter in the two-dimensional quadratic case and it is quasiperiodic in the three-dimensional cubic case.