Abstract to

School on Interactions between Dynamical Systems and Partial Differential Equations - JISD 2023

Barcelona, Spain, July 03 - 07, 2023.

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On the boundedness of solutions of a forced discontinuous oscillator

By understanding the behavior of solutions of discontinuous differential equations, we can gain insight into various phenomena in both natural and engineering domains, where systems exhibit sudden changes. For instance, such knowledge can be applied to model the population growth of species when a new predator or prey is introduced or to model chemical reactions that lead to the formation of solids.

In this work, we perform a qualitative study in the discontinuous undamped oscillator

$$\ddot{x} + \operatorname{sgn}(x) = \varepsilon\, p(t), \tag{1}$$

where sgn stands for the standard sign function, $\varepsilon \geq 0$ is a small real parameter and

$$p(t) = a_0 + \sum_{k\geq 1} (a_k \cos(kt) + b_k \sin(kt))$$

is real analytic and $2\pi-$periodic in t. More specifically, we determine the existence of infinitely many nested invariant tori of (1) with large amplitude, which in turn involves the boundedness all solutions of (1).

The strategy in showing such result is to consider the impact map defined in a subset of the discontinuous set $\Sigma = \{(t, x, \dot{x}) \in \mathbb{R}^3 : x = 0\}$. In good coordinates, this map turns out to be exact sympletic close to an integrable twisting map. Then, by means of the parametrization method, we obtain a

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version of the KAM theorem for a general class of exact sympletic twisting maps that, applied to our case, provides an uniform upper bound for ε that depends only on p. This result allows to determine the persistence of invariant tori carrying quasi-periodic solutions of large amplitude.

Key words: KAM theory, non-smooth systems, invariant tori, parametrization method.