Optimal growth patterns in embryo development and organism locomotion

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The modelling of growth may be applied to either developmental process of organs and contractile biological tissues and organisms. The partial differential equations (PDEs) corresponding to the mechanical equilibrium of biologically active bodies can be established and numerically solved by including concomitant elastic and growing deformations. The amount of growth is however in many instances unknown, and needs to be retrieved from experimental data, or deduced from optimality conditions. We here consider growth patterns for two apparent different systems: i) organ embryonic development and ii) locomotion of soft bodies on frictional substrates.

In the first case we aim to fitting a non-homogeneous and non-isotropic growth that best fits experimental observations. For this, we solve an inverse optimisation problem with discretised PDEs corresponding to the momentum balance of soft elastic bodies. The objective considers the mismatch of the computed and measured deformations and regularsiation terms of the spatial distribution of growth. We apply the methodology to the early development of mouse heart, in order to retrieve potential causes of heart malformations, such as the criss-cross heart, where left and right ventricles are positional in an abnormal vertical position [1]. Figure shows some reconstructed images of the control and mutated hearts between 9.5 and 10.5 days after fertilisation.

Similar PDEs are also employed for describing the active deformation of deformable organisms for self propelling in substrates with non-siotropic friction η. In here, growth is interpreted as an active contraction or extension. In this case, the optimal time dependent growth evolution is found through optimal control theory, where the total deformation $\mathbf{F}$ is decomposed in elatic and growing contributions, and the following constrained minimisation problem is solved:

$$
\begin{aligned}
&\min_{\gamma, x} \left(\int_0^T ||x - x_{target}||^2 + \alpha ||\gamma(x,t)||^2 \right) dt \\
&s.t. \nabla \cdot \sigma(x,\gamma) = 0, \quad \text{in } \Omega \\
&\quad \mathbf{F} = \mathbf{F}^e \mathbf{F}^g(\gamma) \\
&\quad \sigma n = \eta \dot{x}, \quad \text{in } \Gamma_{susbtrate}
\end{aligned}
$$

We solve forward and adjoint equations iteratively in order to find optimal values of the growht $\gamma(x,t)$. The methodology is applied to contact conditions where forward/backward friction, or normal/tangential friction is unequal, leading to distinct locomotion strategies [3]. Figure shows some snapshots of three possible strategies.

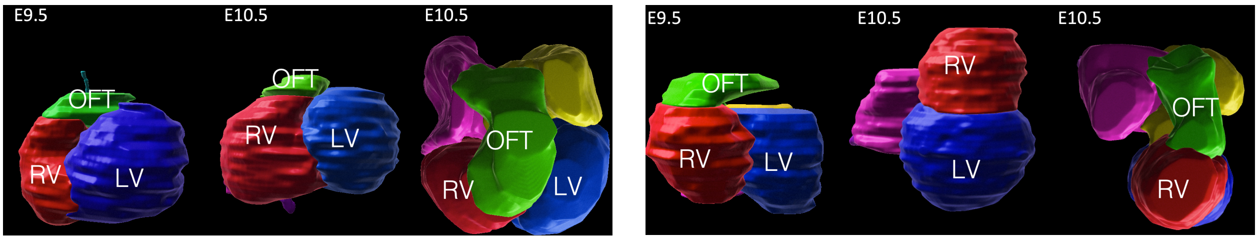

Figure 1: Left: Snapshots of experimentally reconstructed control mouse heart between stages 9.5 and 10.5. Right: Same stages of mutated *crebl1* heart, where ventricles have been twisted. LV,RV=Left and Right Ventricle. OFT=Outflow Track.

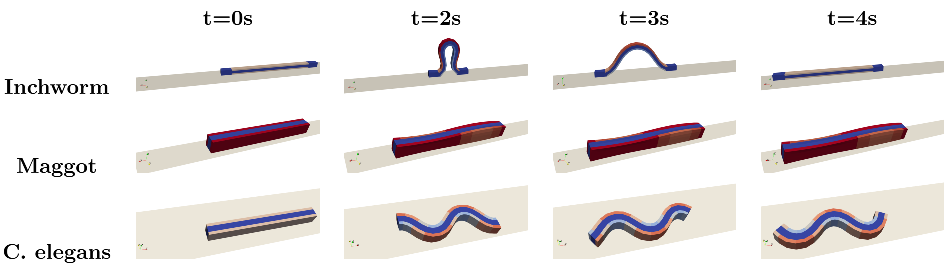

Figure 2: Growth distributions and resulting deformations for non-isotropic fricitonal conditions for three different strategies of slender bodies.

References

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