A computational local reduced-order method for a Rayleigh-Bénard problem

Jesús Cortés[1], Henar Herrero[1], Francisco Pla[1]

Jesus.Cortes@uclm.es

[1] Departamento de Matemáticas, Universidad de Castilla-La Mancha, Ciudad Real (Spain)

Abstract

In this work a local reduced-order method is applied to a 2D Rayleigh-Bénard bifurcation problem, with the Rayleigh number as bifurcation parameter [1]. The Rayleigh-Bénard problem is modeled by the incompressible Navier-Stokes and heat equations. The reduced-order method is based on the offline-online paradigm [2]. On the one hand, in an offline stage, a high-fidelity solver is used to obtain some solutions to the problem for different values of the parameter: the snapshots. The snapshots are processed, using k-means and proper orthogonal decomposition, to obtain several local bases. These bases are local in the sense that they are intended to represent the solutions of the problem in different regions of the solution space. Finally, the original equations of the model are projected onto the local POD bases leading to several local reduced-order problems. On the other hand, in the online stage, while tracing the bifurcation diagram, the reduced-order solver switches intelligently between the different local problems.

The local reduced-order method is used to numerically trace several branches of solutions. The method is benchmarked against a Legendre collocation scheme, and a standard global reduced-order method. The errors between the reduced-order solutions and high-fidelity solutions are of the same order of magnitude. However, in the numerical results, the local reduced-order method is about 2.5 times faster than the global reduced-order method and 400 times faster than the Legendre collocation scheme.

References

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