Shannon information entropy for a N-dimensional quantum nonlinear oscillator

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In this talk I will present some recent results [1] concerning the Shannon information entropy [2] for the quantum version of the Darboux III oscillator. The so-called Darboux III oscillator [3] is an exactly solvable N-dimensional nonlinear oscillator defined on a radially symmetric space with non-constant negative curvature. This oscillator can be interpreted as a smooth (super)integrable deformation of the usual N-dimensional harmonic oscillator in terms of a non-negative parameter λ which is directly related to the curvature of the underlying space.
In particular, analytical results for the Shannon entropy in the position space can be found in the N-dimensional case, and the known results for the quantum states of the N-dimensional harmonic oscillator are recovered in the limit of vanishing curvature $\lambda \to 0$. However, the Fourier transform of the Darboux III wave functions cannot be computed in exact form, thus preventing the analytical study of the information entropy in momentum space.
Nevertheless, the latter can be computed numerically and we find that by increasing the absolute value of the negative curvature (through a larger λ parameter) the information entropy in position space increases, while in momentum space it becomes smaller. This result is indeed consistent with the spreading properties of the wave functions of this quantum nonlinear oscillator, which will be explicitly shown.

References

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[2] C. E. Shannon. *A mathematical theory of communication.* Bell System Technical Journal, **27** (1948) 379–423.

[3] A. Ballesteros, A. Enciso, F. J. Herranz, and O. Ragnisco. *A maximally superintegrable system on an n-dimensional space of nonconstant curvature.* Physica D: Nonlinear Phenomena, **237** (2008) 505–509.