BOUNDARY REGULARITY OF STABLE SOLUTIONS TO SEMILINEAR ELLIPTIC EQUATIONS

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We study the boundary regularity of stable solutions to semilinear elliptic equations. A solution is *stable* if the principal eigenfunction of the linearized equation is nonnegative. For variational problems, stability amounts to the nonnegativity of the second variation, and hence includes the class of (local and global) minimizers.

Given a bounded domain $\Omega \subset \mathbb{R}^n$ of class $C^{1,1}$ and a function $f \in C^1(\mathbb{R})$, we consider stable solutions to

$$\begin{cases} -Lu = f(u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where L is a uniformly elliptic operator of the form

$$Lu = a_{ij}(x)\partial_{ij}u + b_i(x)\partial_i u.$$

Here, the regularity reduces to showing that solutions are bounded, since the linear theory then allows to prove further smoothness properties. This turns out to be a delicate question and depends on the dimension of the space. In [3] we show that stable solutions are globally Hölder continuous when $n \le 9$ for *all* (nonnegative, nondecreasing, convex) nonlinearities. Our result is optimal, since there are examples of unbounded stable solutions for $n \ge 10$.

Before our work, the only optimal result by Cabré, Figalli, Ros-Oton, and Serra [2] for the model operator $L = \Delta$ needed a C^3 regularity assumption on the domain. Moreover, our proofs are all quantitative thanks to a new device of Cabré [1], which has already been applied in [4] to obtain interior estimates. Our bounds depend on the ellipticity constants and on the norms $\|\nabla a_{ij}\|_{L^\infty}$ and $\|b_i\|_{L^\infty}$ of the coefficients. This dependence is crucial to weaken the regularity of the domain.

References

[1] X. Cabré, *Estimates controlling a function by only its radial derivative and applications to stable solutions of elliptic equations*, Preprint at arXiv 2211.13033 (2022),

[2] X. Cabré, A. Figalli, X. Ros-Oton, J. Serra, *Stable solutions to semilinear elliptic equations are smooth up to dimension 9.* Acta Math. **224** (2020) 187-252.

[3] I. U. Erneta, *Boundary regularity of stable solutions to semilinear elliptic equations.* Forthcoming.

[4] ———, *Stable solutions to semilinear elliptic equations for operators with variable coefficients.* Commun. Pure Appl. Anal. **22** (2023) 530-571.