The Hausdorff dimension of planar elliptic measures

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In the plane, in 1988 Jones and Wolff showed $\dim_{\mathcal{H}} \omega_\Omega^p \leq 1$, and in 1993 this result was improved by Wolff by finding a subset $F \subset \partial\Omega$ with σ-finite length and with full harmonic measure $\omega_\Omega^p(F) = 1$.

In the same direction for higher dimensions, $\mathbb{R}^{n+1}$ with $n \geq 2$, Bourgain in 1987 proved that there exists a constant $b_n > 0$ such that $\dim_{\mathcal{H}} \omega_\Omega^p \leq n + 1 - b_n$. Wolff in 1995 constructed a set $\Omega_n \subset \mathbb{R}^{n+1}$ such that $\dim_{\mathcal{H}} \omega_{\Omega_n}^p > n$.

We focus on the study of the dimension of planar elliptic measures arising from the PDE $\mathrm{div}(A\nabla\cdot) = 0$ with uniformly elliptic matrix A. In this scenario, we present the analogous result of Jones-Wolff or Wolff in the plane for these situations:

- Reifenberg flat domains with small constant and Lipschitz matrices.
- The study of planar elliptic measures via quasiconformal mappings.

This is a joint work with Martí Prats and Xavier Tolsa.