Linearizable projective differential equations and integrable two-dimensional Riemannian metrics

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In this talk we consider a family of cubic, with respect to the first derivative, second-order differential equations. This family of equations is a projection of the Hamiltonian system for the geodesics of a two-dimensional Riemannian metric. Consequently, considered family of equations is called a projective equation. Both autonomous and non-autonomous first integrals of the projective equation can be used to establish integrability or superintegrability of the Hamiltonion equations for the geodesics. Here we demonstrate how linearizable via nonlocal transformations projective equations can be used to this aim.

We propose a general procedure of how to construct a superintegrable metric with one linear and one nonlinear, with respect to the momentum, first integrals from an autonomous projective equation. We classify equations that can be used in this construction and demonstrate that they can be linearized via nonlocal transformation. We illustrate this construction by the example of the anharmonic oscillator and demonstrate that it corresponds to a supertintegrable metric with a linear and a transcendental first integrals. At certain values of the parameter the transcendental first integral degenerates into a polynomial one with an arbitrary even degree. We believe that it is a first example of a superintegrable metric with a linear and a transcendental first integrals.

In addition, we consider the equivalence class of the anharmonic oscillator with respect to the point transformations and show that there are nontrivial examples of the Liénard equations that belong to it, like the generalized Duffing oscillator or the generalized Duffing–Van der Pol oscillator. Finally, we discuss applications of non-autonomous linearizable projective equations for the construction of integrable two-dimensional Riemannian metrics.

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