Volcano transition in populations of phase oscillators with random nonreciprocal interactions

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Populations of heterogeneous phase oscillators with frustrated random interactions exhibit a quasi-glassy state in which the distribution of local fields is volcano-shaped [1]. In a recent work [2] the volcano transition was replicated in a solvable model using a low-rank, random coupling matrix **M**. One important limitation of this model is the symmetry of **M**. Interactions are indeed nonreciprocal in a number of contexts.

In this contribution we extend the model in [2] including tunable nonreciprocal interactions, i.e. $\mathbf{M}^T \neq \mathbf{M}$. More precisely, we put forward two solvable models of populations of oscillators with low-ranked, asymmetric, random interactions. Specifically, we introduce a free parameter $\eta \in [-1, 1]$, which allows us to continuously interpolate between fully symmetric ($\eta = 1$) and fully antisymmetric interactions ($\eta = -1$), going over the uncorrelated case ($\eta = 0$). This new ingredient does not degrade the tractability of the models. Moreover, we refine the analysis in [2] and allow the frequency distribution to be any unimodal symmetric distribution (not only Lorentzian). Our numerical simulations fully confirm the analytical results.

Surprisingly, in spite of the similarities of the models introduced here, their phase diagrams turn out to be notably different. For comparison purposes, we carry out simulations with the equivalent model with a full-rank random coupling matrix [1, 3]. We find that the volcano transition is only possible above a critical level of reciprocity, different from the low-rank models. Our work evidences that the failure of the extrapolation from low-rank to full-rank structural disorder, refuting a conjecture raised in [2].

References

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