# Splitting of separatrices in generalized standard maps

**Dídac Gil Rams**, PhD Student at UPC.
**Joint work with** Inmaculada Baldomà Barraca and Pau Martín de la Torre.

**Abstract**

We study transversal intersections between the invariant manifolds (stable and unstable) associated to an hyperbolic fixed point for a class of maps. These intersections are known as homoclinic orbits. The existence of these kind of orbits is one of the most celebrated methods to prove the existence of chaotic dynamics in a system. Indeed the Morse-Smale theorem ensures that if there exist transversal intersections between the invariant manifolds of the same invariant object, the system is locally conjugate to a Smale horseshoe with infinite symbols.

The classical Melnikov theory is a first order perturbative theory that, in addition, can be used to measure the intersection angle between the invariant manifolds. However straightforwardly there are cases where the Melnikov function is exponentially small and the associated theory is not true. In these cases, to measure, for example, the intersection angle between the manifolds, becomes a difficult and technical task, since it is a beyond all orders phenomenon. This is the case of the problem we are considering.

We study the splitting of separatrices on generalized standard maps. This generalization includes the already studied maps like *the standard map*, first studied by Lazutkin, or *the perturbed McMillan map*.

More concretely, we are going to study the intersection of the invariant manifolds associated to a fixed point of the discrete dynamical system

$$\begin{cases} x^* = x + y + f(x,h), \\ y^* = y + f(x,h), \end{cases}$$

where $h$ is a small parameter and $f$ depends analytically on $|h| < h_0$, $|x| < \rho_0$, for some fixed $h_0, \rho_0 > 0$. We consider $f$ to be of the form

$$f(x,h) = \sum_{k \geq 0} f_k(x) h^{k+2},$$

with $f_k(x) = \sum_{j=1}^{d_k} f_{k,j} x^j$ and $f_{k,d_k} \neq 0$. In addition some extra condition on the exponents $d_k$ are imposed.

We obtain an asymptotic formula for the Lazutkin invariant, value related to the area between two homoclinic points, and its first term depends on a Stokes constant that is generically different from zero. To do so, one of the techniques that we use is *the inner equation* related to our generalized standard maps.