TITLE

Critical transitions in nonautonomous ordinary equations with concave derivative

ABSTRACT

A function with finite asymptotic limits gives rise to a transition equation between a "past system" and a "future system". We analyze this situation in the cases of nonautonomous coercive scalar ODEs with concave derivative with respect to the state variable, assuming the existence of three hyperbolic solutions for the limit systems, in which case the upper and lower ones are attractive. The different global dynamical possibilities are described in terms of the internal dynamics of the pullback attractor: cases of tracking of the two hyperbolic attractive solutions or lack of it (tipping) arise. This analysis allows us to present cases of rate-induced critical transitions, as well as cases of phase-induced and size-induced tipping. Our conclusions are applied in models of mathematical biology and population dynamics: we describe rate-induced tracking phenomena causing extinction of a native species or invasion of a non-native one, as well as population models affected by a Holling type III functional response to predation where tipping due to the changes in the size of the transition occurs. In all the cases, we see how the appearance of a critical transition can be understood as a consequence of the strength of the Allee effect.

This is a joint work with Jesús Dueñas and Rafael Obaya.