Europa-induced Overlapping of Secondary Resonances in the 4:3 Jupiter-Ganymede Unstable Resonant Orbit Family

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EXTENDED ABSTRACT

The phenomenon of mean-motion resonance overlapping [1] is known to be crucial for the generation of global chaos and large-scale natural transport in celestial systems, and is useful for low-energy space mission trajectory design. Given the importance of unstable mean motion resonant orbits for space missions, an understanding of their properties is critical. In most prior work (e.g. [2,3]), however, the model used is the planar CRTBP (PCRTBP), which takes only the gravitation of one moon into account. When designing tours of multi-moon systems, though, it is necessary to transition between resonances contained in regions of the phase space which are significantly affected by the gravity of two moons. Thus, at least a restricted 4-body model is needed to study these orbits accurately; one such model is the concentric circular restricted 4-body problem (CCR4BP), where a third large body m_3 is added to the PCRTBP.

In this work, as a case study, we investigate the effect of Europa on the Jupiter-Ganymede unstable 4:3 interior resonant orbits in the Jupiter-Europa-Ganymede CCR4BP. In the Jupiter-Ganymede PCRTBP, where Europa's mass is neglected, this is a one-parameter family of unstable periodic orbits, where the parameter can be thought of as eccentricity or as unstable Floquet multiplier (higher eccentricity corresponding to more unstable orbits). This PCRTBP periodic orbit family extends to eccentricities high enough to cause collision with Ganymede, with unstable multiplier going to infinity. However, in our previous work [4], attempts to numerically continue the higher eccentricity unstable PCRTBP 4:3 periodic orbits by m_3 into the CCR4BP (where they correspond to tori) stopped before m_3 reached Europa's physical mass value m_E; in fact, the highest unstable multiplier whose orbit was successfully continued to the physical system was only 2.458 (under a stroboscopic map on the revolution of m_3).

We believe that we now have a dynamical explanation for this continuation failure. What we have discovered is that the effects of secondary resonances, between the periods of certain 4:3 orbits and the period of the perturbation from Europa, are very strong despite their high order. In summary, in the CCR4BP, secondary resonant islands form inside of the 4:3 orbit family, with each island centered near a previously-existing PCRTBP orbit whose period was exactly resonant with Europa's. We have found very strong evidence that these resonant islands, which occur at various secondary resonances in the 4:3 orbit family, are so large in the Jupiter-Europa-Ganymede CCR4BP that neighboring islands overlap.

To study the effect of the secondary resonances on some high-eccentricity 4:3 orbits, we took the portion of the 4:3 Jupiter-Ganymede torus family with stroboscopic map rotation numbers between 2.0327 and 2.0407, and continued a finely spaced mesh of these orbits by m_3, using very small steps (~m_E /250). We found that even for $m_3 \approx m_E/4$, very wide pendulum shaped gaps centered at rotation numbers $2\pi*11/34=2.0328$ and $2\pi*12/37=2.0378$ appear among the remaining tori, as well as less wide but still significant gaps centered at rotation numbers $2\pi*23/71=2.0354$ and $2\pi*25/77=2.0400$. These rational rotation numbers correspond to secondary resonances. Furthermore, as we increase m_3 further, the gaps grow wider, and tori separating the various secondary resonances decrease in number and then disappear, with tori closer to secondary resonances disappearing first. This very strongly indicates that the secondary resonances overlap in the physical-mass Jupiter-Europa-Ganymede CCR4BP. Future investigations in this phase space region for the Jupiter-Europa-Ganymede system

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should thus focus on computing periodic orbits, librational tori, and separatrices contained within secondary resonances.

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