

Emergent synchronization in pairwise and high-order networks: the role of the spectral dimension

From social interactions to the human brain, networks and their high-order counterpart are key to describing the underlying structure and dynamics of complex systems. While it is well known that network structure strongly affects its function, the role that network topology and geometry have on the emerging dynamical properties of a system is yet to be clarified. In this perspective, the spectral dimension plays a key role since it determines the effective dimension for diffusion and synchronization processes on a network. The spectral dimension encodes the underlying geometrical and topological aspects of a network in a single quantity, and retains information of the underlying geometrical space. Thus, it can serve as a bridge between pairwise and high-order systems. Despite its relevance, a theoretical understanding of which mechanisms lead to a finite spectral dimension, and how this can be controlled, still represents a challenge and is the object of intense research.

Here I will present evidence of the link between the geometry of a network, its spectral dimension, and its emergent diffusion and synchronization properties, contributing to a better understanding of the complex interplay between network structure and dynamics. I will then apply this rationale to a well-known synchronization model, the Kuramoto model for coupled oscillators, and derive a high-order Kuramoto model for topological signals based on the topological properties of simplicial complexes.