A Universal Description of Stochastic Oscillators

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Abstract

Many systems in physics, chemistry and biology exhibit oscillations with a pronounced random component. Such stochastic oscillations can emerge via different mechanisms, for example linear dynamics of a stable focus with fluctuations, limit-cycle systems perturbed by noise, or excitable systems in which random inputs lead to a train of pulses. Despite their diverse origins, the phenomenology of random oscillations can be strikingly similar. Here we introduce a nonlinear transformation of stochastic oscillators to a new complex-valued function $Q_1^*(\mathbf{x})$ that greatly simplifies and unifies the mathematical description of the oscillator's spontaneous activity, its response to an external time-dependent perturbation, and the correlation statistics of different oscillators that are weakly coupled. The function $Q_1^*(\mathbf{x})$ is the eigenfunction of the Kolmogorov backward operator with the least negative (but non-vanishing) eigenvalue $\lambda_1 = \mu_1 + i\omega_1$. The resulting power spectrum of the complex-valued function is exactly given by a Lorentz spectrum with peak frequency ω_1 and half-width μ_1; its susceptibility with respect to a weak external forcing is given by a simple one-pole filter, centered around ω_1; and the cross-spectrum between two coupled oscillators can be easily expressed by a combination of the spontaneous power spectra of the uncoupled systems and their susceptibilities. Our approach makes qualitatively different stochastic oscillators comparable, provides simple characteristics for the coherence of the random oscillation, and gives a framework for the description of weakly coupled oscillators.