Lagrangian betwenness: connecting network theory, dynamical systems and fluid flows

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The study of connectivity in networks has brought insights across many fields ranging from neurosciences to epidemic spreading or climate. One of the classical network measures, betweenness centrality, has demonstrated to be very effective in identifying nodes that act as focus of congestion, or bottlenecks. Outside the network framework, however, there is no obvious way to define betweenness. Nevertheless, the concept of bottleneck is equally present in dynamical systems and in fluid flows as in networks.
By using ideas that relate dynamical systems and network theory [1], a trajectory-based formulation of betweenness, called Lagrangian betweenness [2], has been provided, which is computed in terms of Lyapunov exponents. This extends the concept of betweenness beyond network theory and relates hyperbolic points and heteroclinic connections in dynamical systems to the structural bottlenecks of the flow network associated with it.
I illustrate the use and meaning of the Lagrangian betweenness by identifying bottlenecks in ocean surface flows in the Adriatic Sea or the Kerguelen region in the Southern Ocean. Also, by analyzing plankton abundance data from the Kuroshio region in the Pacific Ocean, we find significant spatial correlations between measures of biological diversity and betweenness, suggesting that ocean bottlenecks act as ecological hot spots.

[1] E. Ser-Giacomi, V. Rossi, C. López, and E. Hernández-García, Flow networks: A characterization of geophysical fluid transport, Chaos 25, 036404 (2015).

[2] E. Ser-Giacomi,A. Baudena, V. Rossi, M. Follows, S. Clayton, R. Vasile, C. López and E. Hernández-García, Lagrangian betweenness as a measure of bottlenecks in dynamical systems with oceanographic examples, Nature Communications 12, 4935 (2021).