

# Reconstruction of phase-amplitude dynamics from electrophysiological signals

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Signals from interacting brain regions display transient synchronization of phases and amplitudes in different frequencies. Commonly, the interaction between regions of the brain is quantitatively described by either analyzing the correlations of amplitudes of the measured signals or by calculating phase-synchronization measures. However, for a complete picture of the interactions it is important to analyze the dynamics of both the amplitude and the phase.

In this work, we present a new method for finding the coupling between brain regions by reconstructing the phase-amplitude dynamics directly from the measured electrophysiological signals. For this purpose, we use the recent advances in the field of phase-amplitude reduction of oscillatory systems, which allow the representation of an isolated oscillatory system as a phase-amplitude oscillator in a unique form using transformations (parametrizations) related to the eigenfunctions of the Koopman operator. By combining the parametrization method and the Fourier-Laplace averaging method of finding the eigenfunctions of the Koopman operator, we developed a novel method of assessing the transformation functions from the signals of the interacting oscillatory systems. The resulting reconstructed dynamical system is a network of phase-amplitude oscillators with the interactions between them represented as coupling functions in phase and amplitude coordinates.

Using synthetic signals generated from several models with known and unknown theoretical phase-amplitude reduced forms, we demonstrate that our method is capable of finding the proper unique dynamic form of these oscillatory systems in the reduced phase-amplitude space.

Our method can be applied to describe any network of interacting oscillators as a dynamical system using signals of the network elements. In particular, to analyze the coupling between distant brain regions using high time resolution signals, such as EEG or MEG. Further simulation and study of the reconstructed dynamical system then enables the construction and investigation of a mathematical model of various neural pathologies and disorders of the brain.