Title: Dynamics in the coupling of two Fitzhugh-Nagumo neurons

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Models built by coupling elementary dynamical units are essential pieces for the modeling of a huge number of phenomena, from neural networks to biological oscillators. Our interest is the study of the dynamics that emerge through the coupling, through simple mechanisms, of systems with also simple dynamics, either stationary or oscillatory (see [1, 2]). In this context, we will consider the following coupling of two Fitzhugh-Nagumo systems:

$$\left\{\begin{array}{rcl} x_1' &=& c(y_1 + x_1 - \frac{x_1^3}{3}) + \alpha_1(x_2 - x_1), \\ y_1' &=& -\frac{1}{c}(x_1 - a + by_1) + \alpha_2(y_2 - y_1), \\ x_2' &=& c(y_2 + x_2 - \frac{x_2^3}{3}) + (\alpha_1 + \varepsilon_1)(x_1 - x_2), \\ y_2' &=& -\frac{1}{c}(x_2 - a + by_2) + (\alpha_2 + \varepsilon_2)(y_1 - y_2). \end{array}\right.$$

where

$$0 < b < 1, \quad c > 0, \quad b < c^2,$$

$\alpha_i \in \mathbb{R}$ and $\varepsilon_i \in \mathbb{R}$. Conditions on the parameters are such that the isolated neuron either exhibits an attracting equilibrium point or an attracting limit cycle, that emerges through a supercritical Hopf bifurcation. The model includes the case of symmetric coupling ($\varepsilon_1 = \varepsilon_2 = 0$), and the asymmetric one, ($\varepsilon_1 \neq 0$ or $\varepsilon_2 \neq 0$). The plane $\Pi = \{(x_1, y_1, x_2, y_2) \,|\, x_1 = x_2,\, y_1 = y_2\}$ is invariant and the constrained dynamics is that of an isolated Fitzhugh-Nagumo system. Note that the attractors of the system that are contained in Π determine the synchronization states of the model. We know that there is a Hopf bifurcation in Π and we wonder about the additional degeneracies that can occur in the directions transversal to such plane. The simplest scenarios lead to codimension two Hopf-pitchfork and Hopf-Hopf bifurcations. Our goal is to describe the variety of cases that can be unfolded by the model and to discuss some of the most interesting dynamical consequences: presence of invariant tori and existence of chaotic dynamics. Similar models have been considered in [3], and more recently in [4].

References

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