Ranking and serial thinking: a geometrical solution

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Serial thinking is a cognitive function underpinning almost any of our daily actions. Our brain continuously encodes and learns ordered sequences that we can remember, process, and replay as in motor planning and language production. Despite the ubiquity of such computation, a complete understanding of the neuronal machinery underpinning this function is still lacking. Psychophysical evidence suggests that the ordering of learned information can be solved by resorting to a linear mental workspace, namely the mental line, in which the relevant items are mapped according to their arbitrarily assigned rank.

Here, we prove that arbitrarily ranking a set of M abstract symbols can be always performed by looking at the projections of their inner representation onto a suited low-dimensional subspace. The subspace where symbols ranks are encoded works as a ``geometrical mental line" (GML), given by a suited linear combination of the symbols representations. The existence of a GML thus implies that the dimension of the representational space reduces

from M to one as a byproduct of the nonlinear dynamics naturally occurring into a recurrent neural network, compressing the information about the ranks into a scalar magnitude. This task solution explains all the behavioral effects and neuronal correlates observed in non-human primates performing an implicit serial learning task, the transitive inference (TI). The derived mental line, related to the widening of the dimensionality of the sensory space,

turns out to be naturally implemented in a recurrent neural network.

Finally, this framework allows us to make predictions about possible animal behavior under yet unexplored task conditions.