Natural eccentricity growth in Medium Earth Orbit due to the lunar perturbation as an Arnold diffusion process

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Among the various actions that are being taken to preserve the circumterrestrial environment, end-of-life disposal solutions play a key role. In this regard, innovative mitigation strategies should be conceived not only following novel technologies, but also advanced theoretical understanding. Recently, several studies have showed that natural perturbations can lead the satellites towards an atmospheric re-entry, without expensive maneuvers, also departing from high-altitude regions.
In the case of the Medium Earth Orbit region, home of the navigation satellites like GPS and Galileo, the main driver is the gravitational perturbation due to the Moon, that can increase the eccentricity in the long term. In this way, the pericenter altitude gets into the atmospheric drag domain and the satellite eventually re-enters.
In this work, we provide a new explanation of the phenomenon, following the Arnold diffusion concept. Focusing on the case of Galileo, we consider a hierarchy of Hamiltonian models, assuming that the motion of the spacecraft is affected by the oblateness of the Earth and the gravitational attraction of the Moon. First, the Moon is assumed to lay on the ecliptic plane and periodic orbits and associated hyperbolic invariant manifolds are computed for various energy levels, in the neighborhood of a given resonance. Along each hyperbolic manifold, the eccentricity increases naturally, reaching its maximum at the first intersection between the stable and the unstable ones. Then the inclination of the Moon is taken with its real value, translating the problem into a non-autonomous one. Under the ansatz of transversality of the stable and unstable manifolds, checked numerically, Poincaré-Melkinov techniques are applied to show how the satellite can move from one energy level to the other, and thus on different hyperbolic manifolds, to eventually reach the value of eccentricity corresponding to the atmospheric re-entry.