Classifiability of transformation group C*-algebras

Eusebio Gardella

Abstract

By the C*-sequence algebra of a C*-algebra A we mean the norm bounded sequences in A modulo those tending to zero. A classical construction shows that the C*-sequence algebra of a II_1 factor has a quotient which is itself a II_1 factor (the W*-ultrapowers). Similarly, any C*-algebra with a tracial state gives rise to a quotient in the C*-sequence algebra which is a W*-algebra. In recent years, such constructions have been used (e.g. by Matui-Sato on Toms-Winter regularity, and Schafhauser's AF embedding theorem) to lift deep results from W*-algebras to C*-algebras (through the C*-sequence algebra). I will mainly survey some of these results (and current work in progress), aGiven a topological dynamical system, there is a standard way to associate a C*-algebra to it, namely the crossed product (also known as the transformation group C*-algebra). Crossed products have provided both new examples of interesting C*-algebras, as well as new ways to study already known ones (such as the irrational rotation algebra). Given the tremendous recent success of the classification program for simple, nuclear C*-algebras, it becomes an imperative task to determine, in dynamical terms, when a crossed product belongs to the classifiable class. I will give an overview of the state of the art, highlighting the surprising differences between the cases of amenable and nonamenable groups. The results on nonamenable groups are joint work with Shirly Geffen, Julian Kranz and Petr Naryshkin.