

Inductive limit descriptions of nuclear C^* -algebras

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Abstract

Inductive limits are a central construction in C^* -theory because they allow one to use well-understood building blocks to naturally construct intricate C^* -algebras whose properties remain tractable. One task for the structural theory of operator algebras is to determine which classes of operator algebras arise as inductive limits of nice operator algebras.

The model result in this direction is Connes' 1970's theorem showing that many classes of von Neumann algebras, including semi-discrete von Neumann algebras, arise as inductive limits of finite dimensional von Neumann algebras. For C^* -algebras, an analogous result fails outright: many nuclear C^* -algebras are not inductive limits of finite dimensional C^* -algebras. However, by generalizing our notion of an inductive system, we can in fact describe any nuclear C^* -algebra as the limit of a system of finite dimensional C^* -algebras. Though seemingly abstract, these generalized inductive systems arise naturally from completely positive approximations of nuclear C^* -algebras. This is based in part on joint work with Wilhelm Winter.