CHAOTIC BEHAVIOR, INVARIANT MANIFOLDS AND EXPONENTIALLY SMALL PHENOMENA IN DYNAMICAL SYSTEMS

Dates
Tuesdays and **Thursdays**
from 17:00h to 19:00h

From **February 1st** (6 weeks)

Location
Aula 103.
Facultat de Matemàtiques i Estadística.
Universitat Politécnica Catalunya.
Carrer de Pau Gargallo, 14,
08028 Barcelona

Description:

It is well know that one cannot compute the solutions of most of differential equations. For this reason, since H. Poincaré, the aim of the qualitative theory of Dynamical Systems is to describe the behavior of the solutions and understand which mechanisms structure this behavior.

A key point in this approach is to analyze the invariant objects that the differential equation may possess and the orbits which connect them: these are the normally hyperbolic invariant manifolds and their stable and unstable manifolds.

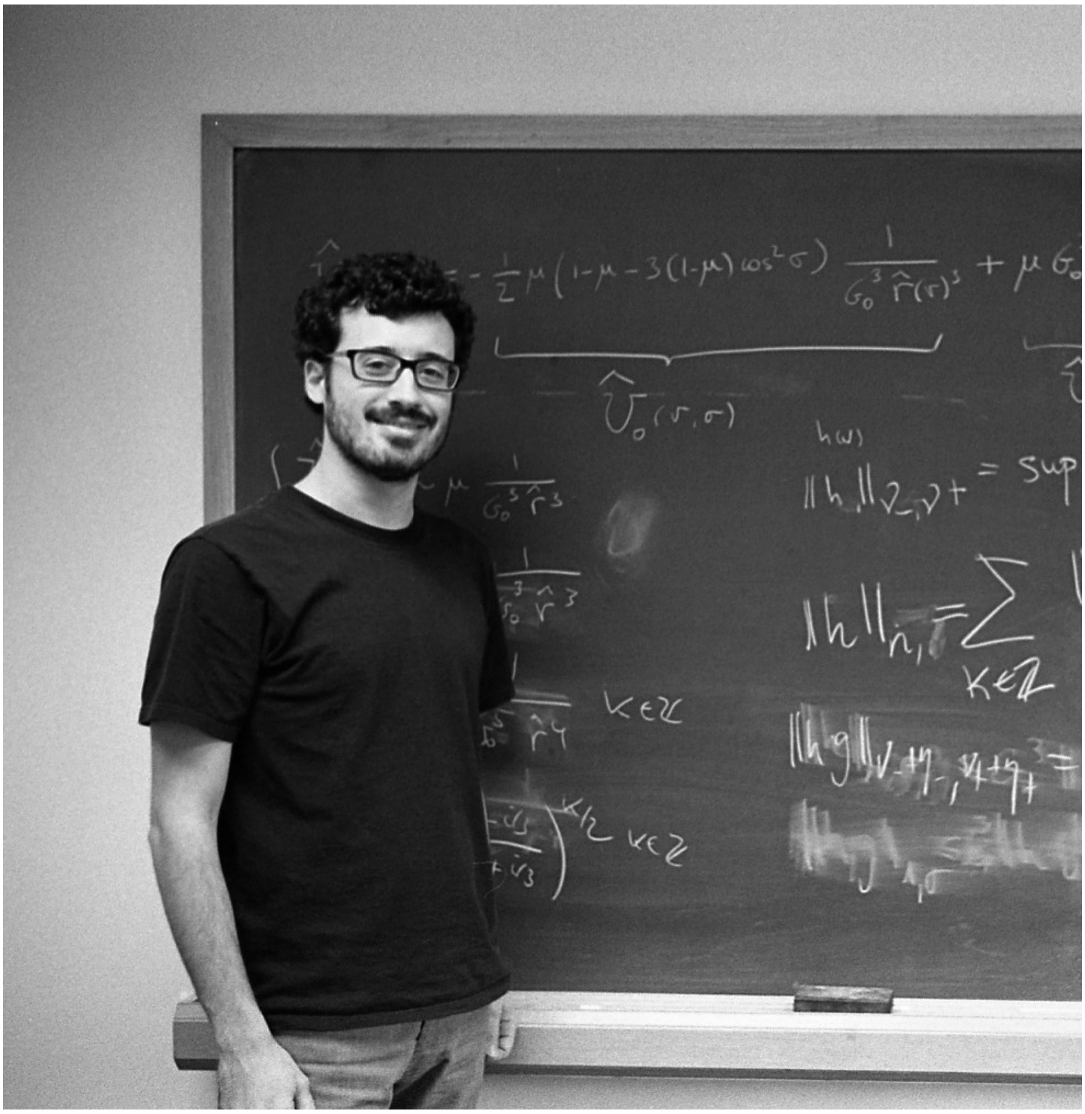
The goal of this course is to understand the role of invariant manifolds and their intersections on the global dynamics of different models. In particular how this intersections give raise to chaotic dynamics and instability phenomena.

In the first part of the course, after presenting the basic theoretical tools in Dynamical Systems, we will show some examples coming from different areas (Celestial Mechanics, Hamiltonian Partial Differential Equations, Bifurcation theory) and describe (some of) their invariant objects and how they articulate unstable motions.

In the second part of the course we will construct the invariant manifolds that these objects may possess and explain how to prove the existence of their intersections using classical perturbation theory. We will also consider examples where the distance between these manifolds is exponentially small with respect to a small parameter and therefore classical perturbative methods cannot be applied to detect their intersection.

Lecturers

It is no more possible to register for this event



BGSMath Organisers

Marcel Guardia (UPC) and Tere Seara (UPC)

Syllabus

1) Classical Invariant Manifold Theory of hyperbolic critical points and periodic orbits.

2) Chaotic dynamics

2.1. The Smale horseshoe and symbolic dynamics

2.2. Transverse homoclinic orbits and the Smale Theorem

2.3. Homoclinic orbits to a saddle focus: The Shilnikov Theorem

3) Perturbation theory: Splitting of separatrices and the Melnikov Method

4) Exponentially small Splitting of separatrices: Examples

4.1. Nearly integrable Hamiltonian Systems of low degrees of freedom

4.2 The rapidly periodically forced pendulum

4.3. Arnold Difusion

4.4. Oscillatory motions in Celestial Mechanics

4.5. Hopf-Zero singularity

4.6. Breathers in Hamiltonian PDEs

5) Melnikov method revisited and the Hamilton-Jacobi equation

6) Exponentially small Splitting of separatrices: Methods

6.1. The regular case: Validity of the Melnikov prediction

6.2. The singular case: The inner equation and the Stokes constant

References

Baldomá, Inma; Guardia, Marcel; Fontich, Ernest; Seara, Tere M. Exponentially small splitting of separatrices beyond Melnikov analysis: rigorous results Journal of Differential Equations, 253 (12): 3304-3439 (2012)

Guckenheimer, John; Holmes, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Applied Mathematical Sciences, 42. Springer-Verlag

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Moser, Jürgen. Stable and random motions in dynamical systems. Annals of Mathematics Studies, No. 77. Princeton University Press

Wiggins, Stephen Introduction to applied nonlinear dynamical systems and chaos. Second edition. Texts in Applied Mathematics, 2. Springer-Verlag

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